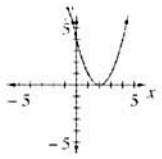
In Class 1.1: Functions, Domain and Range Name: KEY

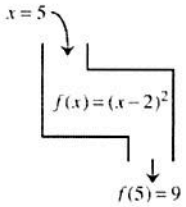
Algebra II, Unit 1: Functions Team Role:

Part 1: Functions and Relations

A relationship between inputs and outputs is a FUNCTION if there is exactly one output for each input. Functions are often written as y = some expression involving x, where x is the input and y is the output. The following is an example of a function. A relation is a function if there are no vertical lines that intersect the graph at more than one point.

x	−2	−1	0	1	2	3	4	5
y	16	9	4	1	0	1	4	9

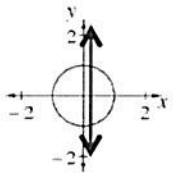




In the example above the value of y depends on x, so y is also called the DEPENDENT VARIABLE and x is called the INDEPENDENT VARIABLE.

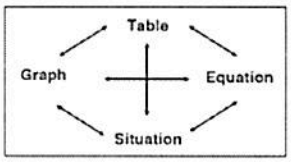
$x^2 + y^2 = 1$ is not a function because there are **TWO y-values (outputs) for some x-values**, as shown on the right. It fails the **VERTICAL LINE TEST.**

x	−1	0	0	1
y	0	−1	1	0



Rule of Four

Functions are generally represented in four distinct ways. A function can be represented by:



Function Notation

Another way to write a function is with the notation **"$f(x)$ ="** instead of "y = ".

The function named **"f" has ouput $f(x)$. The input is x.** In the example at right, $f(5) = 9$. The input is 5 and the output is 9. You read this as, **"f of 5 equals 9."**

Domain and Range

The set of possible values for the input of a function is called the DOMAIN of the function. This set consists of every input value for x for which the function is defined.

The RANGE of a function is the set of possible values of the output. This set contains every y-value that the function can generate. In the example above, notice that you can input any x-value into the equation and get an output. The domain of this function is "all real numbers" because any number can be an input. The outputs are all greater than or equal to zero, so the range is $y \geq 0$. The symbols $-\infty$ and ∞ represents positive and negative **infinity**. They mean that the domain goes on without ending in the positive or negative direction. Infinity is not a number; it is a concept.

* You can have the same output (y's) for different inputs (x's)

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NOTATION FOR DOMAIN AND RANGE:

Examples of Interval Notation	Process to describe Domain and Range
[2, 4]: Numbers **between** and **including** 2 and 4	1. Write starting and ending point separated by a comma.
(3, 25]: Numbers between 3 and 25, including 25 but not 3:	2. Always parentheses with $-\infty$ *and* ∞
(-5, 10): Numbers between -5 and 10 not including -5 or 10	3. Square Brackets next to a number imply the number is included in the domain/range, round brackets imply number is not included in the domain and range.
(-10, 2) U (2, 6): Numbers between -10 and 2 OR between 2 and 6	Not including -10, 2, 6

Examples of Algebraic Notation	Process to describe Domain and Range
$x \geq 7$ $-\infty \leq y \leq \infty$ $x \neq 3$ *and* $x > 2$	1. Use variables, equality and inequality symbols to describe domain and range
	2. $<, \leq, >, \geq, =, \neq$

Examples of Set Notation	Process to describe Domain and Range
For Discrete* Graphs: D: { 1, 2, 3, 4…….}	1. List all inputs or outputs inside squiggly brackets. For example, the set of Natural numbers.
For Continuous* Graphs: R: $\{y \mid -5 \leq y \leq 3\}$ D: $\{x : -5 \leq x \leq 3\}$	2. Inside squiggly brackets, define a variable (using a "|" or ":", read as "such that") and describe it algebraically.

I. Solve the following:

1) "Find f(3)" means to find the output of function f (x) for an input of $x = 3$.

For the function $\mathbf{f(x)} = \frac{1}{x-2}$, find each of the following values.

a) Find f(4). (This means find the output of the function when x = 4.)

$f(4) = \frac{1}{4-2} = \boxed{\frac{1}{2}}$

b) Find x when f (x) = 1. (This means find the input that gives an output of 1.)

$\frac{1}{x-2} = \frac{1}{1}$

$x - 2 = 1$

$+2 \quad +2$

$\boxed{x = 3}$

Check: $f(3) = \frac{1}{3-2} = 1$ ✓

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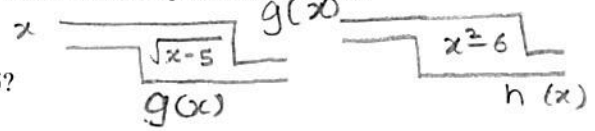
c) What happens to the function when the input is 2? How does it affect the domain of the function?

$f(2) = \frac{1}{2-2} = \frac{1}{0}$ $\frac{N}{0}$ → Function is undefined at $x=2$

This means the domain is: $x \in \mathbb{R}, x \neq 2$

2) Angelica is working with function machines. She has the two machines $g(x) = \sqrt{x-5}$ and $h(x) = x^2 - 6$. She wants to put them in order so that the output of the first machine becomes the input of the second. She wants to use a beginning input of 6.



a) In what order must she put the machines to get a final output of 5?

$h(x) \rightarrow g(x)$

$h(6) = 6^2 - 6 = 30 \rightarrow g(30) = \sqrt{30-5}$

$= \sqrt{25} = 5$

b) Is it possible for her to get a final output of −5? If so, show how she could do that. If not, explain why not.

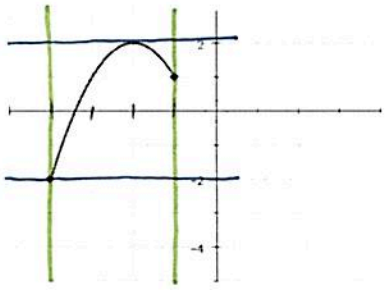
Yes by putting the functions in the order

$g(x) \rightarrow h(x)$ $g(6) = \sqrt{6-1} = 1 \rightarrow h(1) = 1^2 - 6$

$= -5$

Find the domain and range of each relationship. State whether it is a function.

1.

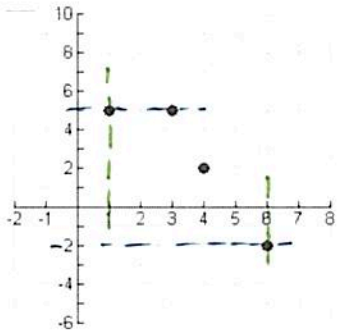


Function? Reason Yes, passes the vertical line test

Domain: [−4, −1]

Range: [−2, 2]

2.



Function? Reason Yes, passes the vertical line test

Domain: {1, 3, 4, 6}

Range: {−2, 2, 5, 5}

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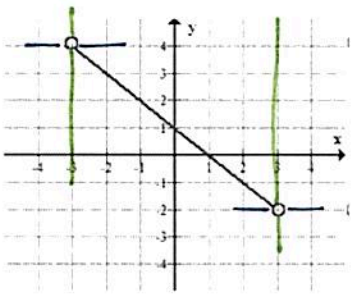
3.



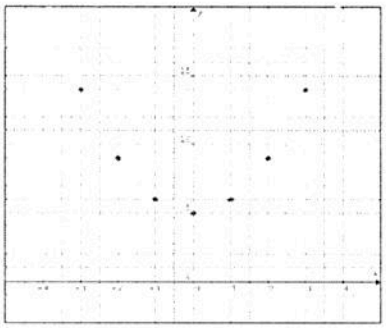
open circle means point is not included

Function? Reason Yes, passes the vertical line test

Domain: $(-3, 3)$

Range: $(-2, 4)$

4.

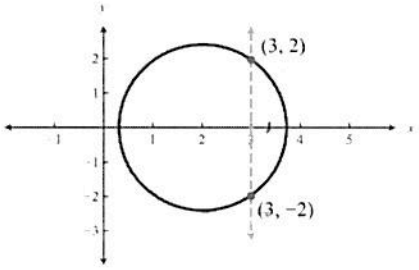


Function? Reason Yes, passes the vertical line test

Domain: $[-3, 3]$

Range: $[5, 14]$

5

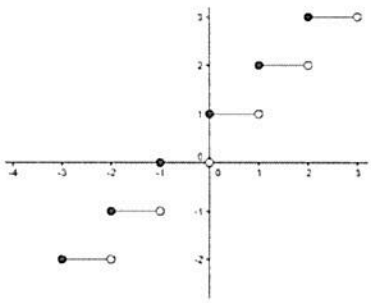


Function? Reason No, fails the vertical line test

Domain: $[1/3, 3\ 2/3]$

Range: $[-2\ 1/3, 2\ 1/3]$

6.



STEP FUNCTION:

Function? Reason Yes, passes the vertical line test

Domain: $[-3, 3)$

Range: $\{-2, -1, 0, 1, 2, 3\}$

Learner Generated Examples

1. Write down two real-life quantities that are related in any way. Describe how they are related.

   Example: Time spent studying and grades on a test. For the most part, the more time you spend studying a particular subject, the better grade you will get. However, if you do not sleep enough, your grade may decline.

Answers will vary.

2. Given the table of values below, change something about this function that would make it a non-function.

X	Y
0	3
1	2
-1	2
2	-1
-2	-1
0	-3

Answers will vary
You could change one of the x values to repeat. Since every input will no longer have a unique output, this will no longer be a function.

Part 2: FUNCTION MACHINES Group Activity

Team Roles – Choose a role and write it down on your paper next to your name.

- **Resource Manager** - If your name comes first alphabetically:
  - Make sure your team has all of the necessary materials, such as the function machines for the warm up.
  - Ask your teacher a question when the *entire* team is stuck. Before raising your hand, you might ask your team, *"Does anyone have an idea? Should I ask the teacher?"*
  - Make sure your team cleans up materials by delegating tasks. You could say, *"I will put away the ____ while you ____."*
- **Facilitator** - If your name comes second alphabetically:
  - Start your team's discussion by reading the question aloud and then asking, *"Which shape should we start with?"* or *"How can we work together to make this shape?"*
  - Make sure that all of the team members get any necessary help. You do not need to answer all of the questions yourself. A good Facilitator regularly asks, *"Do we understand what we are supposed to do?"* and *"Who can answer ____'s question?"*
- **Recorder/Reporter** - If your name comes third alphabetically:
  - Be sure all team members are able to reach the yarn and have access to the resource pages. Make sure resource pages and work that is being discussed are placed in the center of the table or group of desks in a spot where everyone can see them.
  - Be prepared to share your team's strategies and results with the class. You might report, *"We tried ____, but it didn't work, so we decided to try ____."*
- **Task Manager** - If your name comes fourth alphabetically:
  - Remind the team to stay on task and not to talk to students in other teams. You can suggest, *"Let's try working on a different shape,"* or *"Are we ready to try the function machines in a different order?"*
  - Keep track of time. Give your team reminders, such as, "*I think we need to decide now so that we will have enough time to ...*"

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You have a set of four function machines. Your team's job is to get a specific output by putting those machines in a particular order so that one machine's output becomes the next machine's input. As you work, discuss what you know about the kind of output each function produces to help you arrange the machines in an appropriate order. The four functions are reprinted below.

$$f(x) = \sqrt{x} \qquad g(x) = -(x-2)^2$$

$$h(x) = 2^x - 7 \qquad k(x) = -\frac{x}{2} - 1$$

a) In what order should you stack the machines so that when 6 is dropped into the first machine, and all four machines have had their effect, the last machine's output is 11?
b) What order will result in a final output of 131,065 when the first input is 64?

The reporter will report out strategies your team used to solve this problem and whether or not they worked as well as why or why not. The goal is problem solving in teams and points will be awarded for strategizing in teams not the right answer.

TEAM NOTES: ① Cannot start with $f(x) = \sqrt{x}$
Start with $g(x) = -(x-2)^2$ $g(6) = -(6-2)^2 = -16$

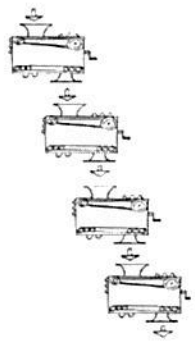
$g(x)$ ↓ $k(x)$ ↓ $h(x)$ ↓ $f(x)$

② Cannot use $f(x)$ or $h(x)$ next so:
$k(6) = -\frac{(-16)}{2} - 1 = 7$

③ Need 11 at the end → $\sqrt{121} = 11$
so: $h(7) = 2^7 - 7 = 128 - 7 = 121$

④ $f(121) = \sqrt{121} = 11$

TEAM PUZZLE



Josh and Grace were working with their team on the function machine problem to put the given function machines in a particular order so that one machine's output becomes the next machine's input. As they worked, they discussed what they knew about the kind of output each function produces to help you arrange the machines in an appropriate order. The four functions are reprinted below.

$$f(x) = \sqrt{x} \qquad g(x) = -(x-2)^2$$

$$h(x) = 2^x - 7 \qquad k(x) = -\frac{x}{2} - 1$$

a) Josh first put an input of 6 into the function $g(x) = -(x-2)^2$ and got an output of -16. He wanted to try $f(x) = \sqrt{x}$ as his next function in the order, but he thinks there might be a problem using -16 as an input. Is there a problem? Explain.

$g(6) = -(6-2)^2 = -16$

$f(-16) = \sqrt{-16}$

Yes, $f(-16)$ is undefined, because we have a negative number underneath a square root. So -16 is not in the domain of $f(x) = \sqrt{x}$ for the set of real #s

b) Because it is not possible to take the square root of -16, it can be said that -16 is not in the domain of the function $f(x)$. The domain of a function is the collection of numbers that are possible inputs for that function. With your team, find two other numbers that are not part of the domain of $f(x)$. Then describe the domain. In other words, what are all of the numbers that can be used as inputs for the function $f(x)$?

$f(x) = \sqrt{x}$ D: $x \geq 0$ or $[0, \infty)$

c) Grace claimed that $g(x)$ could not possibly be the last function in the order for problem 1-2. She justified her thinking by saying, "Our final output has to be 11, which is a positive number. The function $g(x)$ will always make its output negative, so it can't come last in the order." Discuss this with your team. Does Grace's logic make sense? How did she know that the output of $g(x)$ would never be positive? Yes.

$g(x) = -(x-2)^2$ A square of a quantity is always positive. There is a negative sign in front of a square quantity so the final output is negative or 0.

d) Because the outputs of the function $g(x)$ do not include certain numbers, it can be said that positive numbers are not part of the range of the function $g(x)$. The range of a function is the set of all of the possible values that can be outputs. With your team, describe the range of the function $g(x)$. In other words what are all of the values that can be outputs of the function?

$g(x) = -(x-2)^2$ R: $y \leq 0$

CHALLENGE: Extra Credit

1. In what order should you stack the machines so that when 6 is dropped into the first machine, and all four machines have had their effect, the last machine's output is 11?

2. What order will result in a final output of 131,065 when the first input is 64?