**Warm Up:** Identify the type of problem. Write down the steps to solving the following problems. Simplify or solve completely and find the values of x for which the expression or equation is undefined.

1. $\frac{2x}{x^{2}-36}+ \frac{x+4}{x+6}$

2. $\frac{-2x}{x-1}+\frac{x}{3}=\frac{5}{x-1}$

3. $\frac{2x^{2}-7x-4}{x^{2}-9}÷ \frac{4x^{2}-1}{8x^{2}-28x+12}$

A function whose rule can be written as a **ratio of two polynomials,** $f\left(x\right)=\frac{p(x)}{q(x)},$such that $q(x)\ne 0$ is called a \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_.



|  |  |
| --- | --- |
| x | $$f(x) = \frac{1}{x}$$ |
| -2 |  |
| -1 |  |
| $$-\frac{1}{2}$$ |  |
| 0 |  |
| $$\frac{1}{2}$$ |  |
| 1 |  |
| 2 |  |

The parent rational function is:

 $f\left(x\right)= \frac{1}{x} $

Its graph is a hyperbola, which has two separate branches. A rational function may have horizontal and vertical asymptotes. The function $f\left(x\right)=\frac{1}{x}$ has a vertical asymptote at $\\_\\_\\_\\_\\_\\_\\_\\_\\_\\_\\_\\_$ and a horizontal asymptote at $\\_\\_\\_\\_\\_\\_\\_\\_\\_\\_\\_\\_\\_\\_.$

**Transformation of Rational Functions**

Rational Functions can be transformed as follows: $g\left(x\right)= \frac{a}{(x-h)}+k$

Describe the tranformation and graph the following. Identify the domain, range and asymtotes.





1. 2. $f\left(x\right)= \frac{1}{x+2}$ $f\left(x\right)= \frac{1}{x-3 }+2$

Transformations: Transformations:

Vertical Asymptote: Vertical Asymptote:

Domain: Domain:

Horizontal Asymptote: Horizontal Asymptote:

Range: Range:

**Exploration:**



**Part 1:** Given, $p\left(x\right)= x^{2}+x-6, q\left(x\right)=x+2 $

$$h\left(x\right)= \frac{p(x)}{q\left(x\right)}=$$

1. Factor the numerator to find the zeros of $p\left(x\right).$

Factor the denominator to find the zeros of $q\left(x\right)$.

2. Graph the function $g\left(x\right)$ using your graphing calculator. What do you notice about the roots of $p\left(x\right) and q\left(x\right)$ based on the graph?

Zeros of $h\left(x\right)$ are: y-intercept:

Vertical Asymptotes of $h\left(x\right)$ are:

Domain:

Range:

**Part 2:** Now graph $g\left(x\right)= \frac{q(x)}{p\left(x\right)}=$



1. Graph the function $g\left(x\right)$ using your graphing calculator. What do you notice abouit the roots of $p\left(x\right) and q\left(x\right)$ based on the graph?

Zeros of $g\left(x\right)$ are: y-intercept:

Vertical Asymptotes of $g\left(x\right)$ are:

Domain:

Range:

**Horizontal Asymptotes:**



**Identify zeros and any asymptotes or holes for each function.**

1.$f\left(x\right)=\frac{2x+6}{4x-8}$2.$f\left(x\right)= \frac{x-1}{x^{2}-9}$3.$f\left(x\right)= \frac{x^{2}-4}{x-2}$

4. $ f\left(x\right)= \frac{2x^{2}}{x^{2}-x-6}$ 5.$ f\left(x\right)=\frac{x^{2}+1}{x-2}$

**Exploration II: Use your graphing calculator to graph the functions below. Identify the zeros, vertical and horizontal asymptotes, domain and range.**



$$1. h\left(x\right)= \frac{3x-9}{x+3}$$

Zeros: y-intercept:

Vertical Asymptote:

Domain:

Horizontal Asymptote:

Range:

$2. h\left(x\right)= \frac{3x^{2}+3x-6}{x^{2}-x-6}$



$$ $$

Zeros: y-intercept:

Vertical Asymptote:

Domain:

Horizontal Asymptote:

Range:



$$3. g\left(x\right)=\frac{x^{2}+2x-3}{x^{2}-x-2}$$

Zeros:

Vertical Asymptote:

Domain:

Horizontal Asymptote:

Range:

**SUMMARY:**

The process for graphing rational functions of the form: $f\left(x\right)=\frac{p(x)}{q(x)}$ is:

1. Identify the \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_. Find the y-intercept: $f\left(0\right).$
2. Identify the holes, \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_and \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ asymptotes.
3. Plot the \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ and the \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_.
4. Use a table of values to identify additional \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ on the function’s graph.
5. Draw each branch with a \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_.